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BOOK OF ABSTRACTS

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A Tree-Level Hierarchical Agent-Oriented Model for Forecasting the State and Optimizing the Management of the Production Potential of the Region

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This article discusses the processes of predicting the state and optimizing the management of the production potential of the region in the presence of the possibility of using a hierarchical agent-oriented control system. It is proposed to use a deterministic economic-mathematical model for the processes under consideration, in which the dynamics of the main factors (phase vectors) characterizing the state (potential) of industrial objects of the region as a whole, the municipalities that form it and specific industrial enterprises with a three-level spatial structure, is described by the corresponding vector linear discrete-time recurrent equations in the presence of control actions (controls). In the proposed management system, three levels of management decision-making are distinguished – the dominant (regional or first level of management), which is at the disposal of a generalized regional agent, the first subordinate (municipal or second level of management), which is at the disposal of municipal agents, and the second subordinate (the level of manufacturing enterprises or the third level of management), which is at the disposal of production agents. The control system is hierarchical, in which the choice of control of a generalized regional agent determines the resource capabilities of management of municipal and production agents, and the choice of control of a specific municipal agent determines the resource capabilities of control of the corresponding production agents. All agents of the proposed control system are a priori united by certain information and management links. The industrial potential of the region as a whole, its constituent municipalities and industrial enterprises is estimated by the corresponding indicators (linear objective functions) and all agents of the control system are interested in their maximum values. The paper presents formalizations and general schemes for solving the problems of predicting the state and optimizing the management of the region's production potential. The results obtained in the article can be used in the development and creation of intelligent computer systems for information support and management decision support for regional administrations.

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Innovative Monte Carlo Algorithm for Linear Equations Based on "Walk on Equations" Algorithm

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Many scientific and engineering applications are based on the problems of solving systems of linear algebraic equations. For some applications it is also important to compute directly the inner product of a given vector and the solution vector of a linear algebraic system. It is also very important to have relatively cheap algorithms for matrix inversion. The computation time for very large problems, or for finding solutions in real-time, can be prohibitive and this prevents the use of many established algorithms. Monte Carlo methods are methods of approximation of the solution to problems of computational mathematics, by using random processes for each such problem, with the parameters of the process equal to the solution of the problem. The method can guarantee that the error of Monte Carlo approximation is smaller than a given value with a certain probability.

An updated version of Monte Carlo algorithm for solving systems of Linear Algebraic equations is presented and studied. The algorithm is based on the "Walk on Equations" Monte Carlo method recently developed by Ivan Dimov, Sylvain Maire and Jean Michel Sellier. The algorithm is improved by choosing the appropriate values for the relaxation parameters which leads to dramatic reduction in time and lower relative errors for a given number of iterations. A theorem for the convergence of the algorithm has been proved. It is shown that the original algorithm can be optimized if we manage to balance the iteration matrix. Also a sequential Monte Carlo method of John Halton based on on an iterative use of the control variate method has been applied. Numerical tests are performed for examples with matrices of different size and on a system coming from a finite element approximation of a problem describing a beam structure in constructive mechanics.

Acknowledgement. This study is supported by the Bulgarian National Science Fund under Project KP-06-M32/2–17.12.2019 "Advanced Stochastic and Deterministic Approaches for Large-Scale Problems of Computational Mathematics" and Project KP-06-N52/5 "Efficient methods for modeling, optimization and decision making."

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Energy Stable Interior Penalty Discontinuous Galerkin Finite Element Scheme for the Growth Cahn-Hilliard Equation

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We devise and analyze a symmetric interior penalty discontinuous Galerkin finite element method for a growth Cahn-Hilliard type equation with general nonlinear mass term, equipped with essential boundary conditions, viewed as a gradient flow of an energy functional. The proposed scheme is shown to be first order convergent in time. We prove that our scheme is energy stable with respect to a discrete analogue of the continuous free energy of the system, unconditionally in time, and mildly conditioned only with respect to the spatial discretization parameter. We present numerical tests demonstrating the established theoretical results and the robustness of our scheme.

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A Novel Algorithm for Optimization of the Parameters of the Halton Sequence

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The Halton sequences are one of the oldest family of low-discrepancy sequences, widely used in various quasi-Monte Carlo methods. Since long time ago it has been observed that in their original form they suffer from visible artefacts and undesirable correlations between consecutive dimensions. That is why many modifications to their construction were proposed and studied. Since there is sufficient level of freedom within these constructions, for example starting from the Faure's generalized construction, it is possible to design modified Halton sequences which are usable in a wide area of applications, even when the dimensionality of the problem becomes relatively high.

In this work we concentrate on development of a numerical algorithm, which can produce parameters that fully determine instances of the modified Halton sequences which are usable in a setting where the number of dimensions is in the order of hundreds or thousands, while the number of terms of the sequence is

in the same order as the number of dimensions. The optimization algorithm has been implemented to make substantial use of the capabilities of the modern GPUs. We present and discuss numerical results, demonstrating the effectiveness of our approach, when compared with some alternative implementations, as well as the fields of their application.

The proposed modified Halton sequence shows superior behavior for complicated high dimensional problems.

Acknowledgments. This work has been supported by the Project No 30/29.06. 2021, funded by the CAF America Foundation.

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Fuzzy Order Statistics in Reliability Analysis

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We consider an application of fuzzy order statistics introduced in [1]. The lifetimes of components of the system are expressed in terms of the fuzzy order statistics. The reliability and mean residual life functions of sensitive systems are investigated.

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Strong Maximum Principle for Quasi-Monotone Systems of Fully Non-linear Parabolic Equations

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This talk concerns the validity of strong interior and boundary maximum principles for quasi-monotone systems of parabolic equations with fully non-linear principal part.

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The Liouville-Bratu-Gelfand Problem

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We review the classical Liouville-Bratu-Gelfand problem and discuss its generalizations. We also present some recent results.

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Quantification of Uncertainty in Nonlinear Vibrations of Mechanical Systems

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A consensus on the requirement to include key uncertainties in nonlinear dynamical analyses is gradually reached in the research community. Indeed, the uncertainties are inescapable in engineering nonlinear mechanical systems due to a variety of reasons, such as manufacturing and assembling errors, service environment evolutions and external unexpected disturbances. The propagation of uncertainties in nonlinear systems is extremely challenging and computationally expensive due to the deep interactions between nonlinearities and uncertainties. To alleviate the dilemma, the non-intrusive surrogate modeling techniques and efficient nonlinear solution schemes are developed to solve the uncertain nonlinear systems. Results are validated by the traditional crude sampling-based method. Discussions and summarizations are provided for future research.

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Runge–Kutta Based IMEX Scheme for the Kuramoto–Sivashinsky Equation

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This work is centered on the numerical implementation of fourth-order Runge-Kutta based implicit-explicit scheme in time along with compact fourth-order finite difference scheme in space for the solution of one-dimensional Kuramoto-Sivashinsky equation (KSE). The proposed scheme requires solving backward Euler-type linear systems, but only two per time step to get the solution. It is made

possible by employing a novel combination of methods of line (MOL) and partial fraction decomposition techniques. Performance and applicability of the scheme are investigated by testing it on some test examples and by comparing numerical results with relevant known schemes. The numerical results showed that the proposed scheme is more accurate and reliable than existing schemes to solve the KSE.

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Stochastic Default Intensity XVA Pricing Using a PDE Model

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Adjusting derivative prices to take into account default risk has attracted the attention of several researchers and practitioners, especially after the 2007-2008 financial crisis. We derive a novel partial differential equation (PDE) model for derivative pricing including the adjustment for default risk, assuming that the default risk of one of the counterparties (the buyer) follows a Cox-Ingersoll-Ross (CIR) process, while the other party has constant default risk. The time-dependent PDE derived is of Black-Scholes type and involves two "space" variables, namely the asset price and the buyer default intensity, as well as a nonlinear source term. The numerical solution of the PDE is based on finite differences, and a penalty-like iteration for handling the nonlinearity in the case of European derivatives, while a double-penalty iteration is used for American derivatives. We also develop a novel asymptotic approximation formula for the adjusted price of derivatives, resulting in a very efficient, accurate, and convenient for practioners formula. We present numerical results that indicate stable second order convergence for the 2D PDE solution in terms of the discretization stepsize, and at least order 1.5 for the asymptotic solution in terms of the inverse of the mean reversion rate. We compare the effectiveness of the 2D PDE and asymptotic approximations. We study the effect of various numerical and market parameters to the values of the adjusted prices and to the accuracy of the computed solutions.

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Nonstationary Waves in Functionally Graded Viscoelastic Materials

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Continuous spatial material inhomogeneity of materials can occur by purpose during the product manufacturing process, as well as a result of exposure to temperature, corrosion, radiation, and other factors. In this regard, there is a growing interest in studying the behavior under various external conditions of materials with continuously graded properties. This work is devoted to the study of non-stationary wave processes in bodies consisting of functionally graded materials obeying viscoelastic linear integral Boltzmann-Volterra constitutive equations. The aim of the work is to extend the method previously used by the first author in solving non-stationary dynamic problems to solve similar problems for functionally graded materials. The method consists in replacing the original body with continuous material inhomogeneity with a corresponding piecewise homogeneous body build up of a large number of homogeneous components with material parameters set to approximate the continuously graded material parameters of the original body. This method is that it allows using already available analytical solutions of dynamic problems for piecewise homogeneous bodies to investigate the dynamic behavior of functionally graded materials. Note that in the study of non-stationary wave processes in functionally graded materials, even within the framework of linear elasticity, analytical and semi-analytical methods are rarely used and only for a very limited class of problems. Propagation of non-stationary longitudinal waves in viscoelastic continuously inhomogeneous layer and hollow cylinder whose viscoelastic properties gradually vary along the radius are considered. According to the proposed method, the continuously inhomogeneous layer and the cylinder are replaced by the corresponding piecewise homogeneous bodies, for which the solution is constructed. Wave processes in the layer and the cylinder are studied for specific types of continuous inhomogeneities.

Acknowledgements. This study was performed within the bilateral project funded by the Russian Foundation for Basic Research (RFBR), project number 20-58-18002 and by the Bulgarian National Science Fund, project number KP-06-Russia/5 from 11.12.2020.

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Continuous and Statistical Modeling of the Taylor-Couette Flow of Rarefied Viscous Gas with Inhomogeneous Boundary Conditions

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The cylindrical Pirani gauge sensitivity under different conditions is studied in our previous paper. We are modeled the Pirani gauge as cylindrical Couette flow – one dimensional axis symmetrical problem by using a statistical method – Direct Simulation Monte Carlo (DSMC) and the Navier-Stokes-Fourier equations numerical solving. The purpose of this article is to study the cylindrical two-dimensional Taylor-Couette flow with finite length of rarefied gas. The inhomogeneous temperature and velocity profile of the inner cylinder wall (the fiber) is used to modeling the boundary conditions. The finite volume method and statistical modeling is used to study this problem. The realization of such conditions is difficult technically feasible in some cases, but their set in mathematical model gives more opportunities to study the stability in the gaseous medium between the cylinders.

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Kaczmarz Anomaly in Tomography Problems

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Kaczmarz method is an important tool for solving large sparse linear systems that arise in Computational Tomography. The Kaczmarz anomaly phenomenon has been observed recently when solving certain types of random systems. This raises the question of whether a similar anomaly occurs in tomography problems. One aim of this paper is to answer this question. To examine the extent of the phenomenon and to explain its reasons. Another tested issue is the ability of random rows shuffles to sharpen the anomaly and to accelerate the rate of convergence. The results add important insight into the nature of Kaczmarz method.

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Parametric Inference for Geometric Lifetimes of Elements of k-out-of-n Systems

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So called k-out-of-n systems form an important class of systems studied in reliability theory. These systems consist of n elements and work as long as at least k of the elements function. Since such technical structures have some redundancy they find various applications in engineering when highly reliable products are needed.

In my talk I will consider maximum likelihood (ML) estimation of unknown parameters of distributions of component lifetimes of a k-out-of-n system. I will focus on the case when the component lifetimes are discrete and independent random variables. First I will present regularity conditions under which the ML estimators of interest exist almost surely for all sufficiently large n and are strongly consistent. Next I will concentrate on the case when component lifetimes are geometrically distributed and will give a closed-form formula for the ML estimator in this case. Moreover, restricting the attention to n-out-of-n systems I will be able to derive some finite-sample properties of the ML estimator of the geometric parameter. In particular I will obtain its bias and mean squared error.

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New Methods for Analytical Calculation of Elliptic Integrals Applied in Various Physical Problems

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A short review will be made of elliptic integrals, widely applied in GPS communications (accounting for GRT-effects), cosmology, Black holes physics, General Relativity physics and celestial mechanics.Then an analytical method for calculation of zero-order elliptic integrals in the Legendre form will be presented, based on the combination of several methods in the theory of elliptic functions: 1. The recurrent system of equations for higher-order elliptic integrals in two different representations; 2. Uniformization of four-dimensional algebraic equations by means of the Weierstrass elliptic function; 3. A variable transformation, inversely (quadratically) proportional to a new variable.

The developed method rejects the widely accepted belief that elliptic integrals can be calculated only numerically and not analytically.

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Existence and Multiplicity of Positive Solutions for a Third Order Differential Equation

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The aim of this paper is to study the existence and multiplicity of positive solutions of a third order two-point boundary value problem with parameter dependence. The exact expression of the corresponding Green's function is obtained. The results are based on Krasnosel'skii's fixed point theorem in suitable cones. Some examples are given to illustrate the main results.

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Simulation of Rarefied Gas Flows, Comparison with Experimental Data II

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High fidelity modeling and simulations methods need to be anchored to data collected from selected flight tests or experiments to develop robust, accurate and validated supersonic flow-simulation methods to predict the behavior of flowfield throughout the wide range flight regimes including highly rarefied gas flows.

We have developed a unified flow model for compressible flows, based on the Generalized Hydrodynamic Equations (GHE) by Alexeev in 2004, derived from generalized Boltzmann kinetic equation [1]. The model is supposed to account for kinetic effects (intermediate Knudsen number, fluctuations) in the continuum approximation. This model has been explored for simulations of incompressible viscous flows for a wide range of problems and flow parameters, including high Reynolds numbers flows with thin boundary layers, demonstrating good agreement with experimental data [2].

Simulations of compressible supersonic flows is a very challenging problem as such flows can exhibit both continuum and non-continuum flow regimes. Typically, the flow can be continuous to transitional in the near field flow structure, and free molecular flow in the far field. The shock wave (bow shock) is detached from the vehicle at high altitude, and near boundary slip-flow is typical for such regimes. First results for this model has been reported in [3,4].

In this paper we provide a comparison of simulation results of the model (called RNS, the Regularized Navier-Stokes) with the experimental data for rarefied hypersonic flows II, and biconic flow [6,7, 8]. Simulations by DSMC method are also provided, the results by the open source SPARTA DSMC code [6]. The Navier-Stokes model results are provided for the comparison too.

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Optimal Wavelets for Single and Multi-Frequency Signals in Wavelet Based Communications

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Progress in digital wireless communications resulted in extended use of mobile internet access, digital radios, automated highways and factories. However, with the increasing use of wireless services, the requirements on resources like battery power and radio spectrum are put under severe pressure.

Therefore, the development of radio platforms that optimize the utilization of energy in addition to guaranteeing spectral efficiency becomes of great importance. Temporally and spectrally localized transmission strategies that minimize the energy spent to transmit the information-bearing symbols will be crucial towards achieving high energy efficiency [1].

Existing wireless systems are based on the mathematical precept of Fourier transform. The theory of wavelets offers many advantages for the design of wireless communications. The main property of wavelets for these applications is their ability to characterize signals with adaptive time-frequency resolution [2].

In his work, we present the development of wavelets that offer an exact representation of single-frequency and multi-frequency communications. Then, we provide an approach to wavelet optimization that minimizes the power to transmit the signal. Directions to the design of finite band wavelets are discussed.

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Parameter Identification Analysis of the Transfer of Heavy Metals in Honey Bees

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While heavy metals naturally occur in the environment, the various anthropogenic activities have led to their high concentrations in many areas. Such excess concentration has negative effects on the species and human health and may eventually cause total disruption of the natural ecosystems. The level of toxicity of the heavy metals is determined by various factors such as the concentration of heavy metal ions, the population in the area, the environment itself and other. Although the plant growth could be suspended from the heavy metals accumulated in the soil, some plant species could absorb a large amount of heavy metals thus transferring it to the animals and humans. In particular, via pollination, the honey bees receive the heavy metal mass from the nectar and pollen, and it is transferred to honey as well. The real-world experiments are extremely time-consuming, expensive and difficult to manage. This article presents a numerical modeling and study of a kinetic model for the transfer of heavy metal in honey bees by means of plant and soil samples. The paper solves an inverse coefficient identification problem of finding the values of the parameters which are not directly observable but vital for the further investigation, control and prevention of the heavy metal pollution.

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On Modeling the Soliton Interactions of NLS with Vanishing and Non-Vanishing Boundary Conditions

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The dynamical behavior of the N-soliton Manakov trains of NLS model perturbed by external potential terms $V(x)$:

$$i\frac{\partial \vec{u}}{\partial t} + \frac{1}{2}\frac{\partial^2 \vec{u}}{\partial x^2} + (\vec{u}^{\,\dagger}, \vec{u})\vec{u}(x,t) + V(x)\vec{u}(x,t) = 0, \tag{1}$$

has been successfully modeled by the complex Toda chain (CTC), see [1,2]. Here we will briefly review our results on adiabatic soliton interactions as well as the advantages the CTC provide in studying the effects of perturbations of different types on the soliton interactions.

Our next aim now is to extend our approach also to the soliton interactions of dark NLS solitons:

$$\begin{aligned} &i\frac{\partial v}{\partial t} + \frac{1}{2}\frac{\partial^2 v}{\partial x^2} - (|v(x,t)|^2 - \rho^2)v(x,t) + V(x)v(x,t) = 0, \\ &\lim_{x\to\pm\infty} v(x,t) = \rho e^{i\theta_\pm} \end{aligned} \tag{2}$$

where $V(x)$ is an external potential. We will outline the difficulties that the non-vanishing boundary conditions (NVBC) put in the way of such studies.

The first one is that the phase space of equation (2) is a nonlinear one [3]. More precisely $\mathbf{\Phi} = \oplus_\theta \mathbf{\Phi}_\theta$ where θ is a topological integral of motion. In the case of a soliton solution $v_j(x,t)$ its θ_j determines the soliton amplitude. So the first difficulty is to properly introduce the notion of a soliton train $v_{\rm Ns} = \sum_j v_j(x,t)$ which satisfies the boundary condition in (2). The next difficulty is to regularize the action functional so that the integrals become convergent. Finally we have to insert the soliton train into the regularized action functional and to perform the integration over x with adiabatic precision. This will give us an effective action $\mathbf{\Phi}_{\rm Ns}(\theta_j, x_{0j})$ where x_{0j} is the center of mass of j-th soliton, which will provide us the dynamical system for our N-soliton train with $\theta = \sum_j \theta_j$.

At the same time the dark solitons have only one degree of freedom, so one may expect that their interaction would be modeled by the standard (real) Toda chain. The effect of the external potential would lead to perturbations of the Toda chain.

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Bounds on Expected Lifetimes of k-out-of-n Systems with the Decreasing Reversed Failure Rate Components

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We present the optimal upper bounds on expected values of order statistics based on the decreasing reversed failure rate distributions (DRFR). Our results can be applied for precise evaluations of the lifetimes of the classic k-out-of-n reliability systems composed of n elements with i.i.d. IRFR lifetimes $X_1, ..., X_n$, since such systems lifetime can be represented by an appropriate order statistic.

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On Real Hamiltonian Forms of Affine Toda Field Theories

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We will present real Hamiltonian forms of 2-dimensional Toda field theories related to exceptional simple Lie algebras [1], and the spectral theory of the associated Lax operators. Real Hamiltonian forms [2] are a special type of "reductions" of Hamiltonian systems, similar to real forms of semi-simple Lie algebras. Examples of real Hamiltonian forms of affine Toda field theories related to exceptional complex untwisted affine Kac-Moody algebras will be presented.

Along with the associated Lax representations, we will also discuss the relevant Riemann-Hilbert problems and derive the minimal sets of scattering data that determine uniquely the scattering matrices and the potentials of the Lax operators.

This is a joint work [3] with Vladimir Gerdjikov and Alexander Stefanov.

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Main Results and Current Progress within the Scale Invariant Vacuum Paradigm

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A review of the Scale Invariant Vacuum (SIV) idea will be presented as related to Weyl Integrable Geometry [1]. The main results related to SIV and inflation [2], the growth of the density fluctuations [3], and the application of the SIV to scale-invariant dynamics of Galaxies, MOND, Dark Matter, and the Dwarf Spheroidals [4] will be highlighted.

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New Features in the Service for Estimating the Brown Bear (*Ursus arctos L.*) Population in Bulgaria

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The brown bear (*Ursus arctos L.*) in Bulgaria is a strictly protected species ?ccording to the Biodiversity Act. That is why monitoring of the populations of this protected species is necessary for effective management and conservation of their habitats. The monitoring is carried out every year in Bulgaria.

In this work, we present the new features incorporated in the service named ArctosPop for automatic estimation of the brown bear (*Ursus arctos L.*) population in Bulgaria. The program software integrates statistical algorithms, which use as input data the observed data of traces of brown bears during National monitoring. The main and new features of the software service are presented and the results of the brown bear population in Bulgaria computing for the data from the last few National monitorings.

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Keywords: Statistical estimation, Data analysis, Brown bear population

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Reaction Network of a Predator-Prey Model with SEIR and SEIRS Epidemic in the Prey population

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A predator-prey model with an epidemic model of SEIR (Susceptible-Exposed-Infected-Recovered) and SIERS (Susceptible-Exposed-Infected-Recovered-Susceptible) in the prey population is introduced (for details see [1-7]). The epidemic disease has been described by SEIR and SEIRS has been described by a logistical growth function only in the susceptible prey population. The reaction function is given by the Lotka-Voltaire model which consists of five nonlinear ordinary differential equations describing the population of the species. The main focus of this article is that the proposed eco-epidemiological model can be implemented by a reaction network. The studied reaction net-work is a mathematical-chemical apparatus through which a parallel between the change of reactants concentrations and the dynamics of the populations can be made.

The authors present a reaction network, which describes the considered model indicating the obtained differential equations and using the Law of Mass Action Kinetics. Some numerical experiments that show this concept will be given.

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Formation Conditions of the Stratified Fluid Streak from the Spots Chain with Intervals

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The problem of the spots chain dynamics of mixed liquid in stratified environment where spots are arranged horizontally at some interval from each other is considered. The problem is solved in a unique rectangular area with flow periodicity boundary conditions on the left and right boundaries. Under the influence of hydrodynamic forces, this chain of radius 1 turns into the streak of the width of $\pi/2$. We study the formation conditions of a streak of stratified fluid depending on the interval length in the spots chain. Three stages of the spots chain collapse have been studied. At the first stage, the spot is flattened and takes the form of a curved streak, while the layers of salinity perturbation do not mix. At the second stage, the spot makes long oscillatory movements. At the third, longest stage, the oscillations die out and the streak becomes horizontal and continuous. Salinity is chosen as a stratifying component. The problem is described by the Navier–Stokes equations in the Boussinesq approximation. To solve the problem, the SMIF method (Splitting Method for Incompressible Fluid) is used, of course, the finite different scheme of which has such properties as the second order of approximation on spatial variables, minimal scheme dissipation and dispersion, performance in wide range of Reynolds and Froude numbers and when solving wave processes are a property of monotony. This task is of interest both to theorists in terms of the theoretical hydrodynamics of stratified liquid, and experimenters, in terms of the time of the establishment of a stationary field.

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Relativistic Look of a Dark Energy Star

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We illustrate light deflection near ultra-compact neutron stars possessing a photon sphere motivated by modern confirmations of general relativity. Adopting the phantom scalar field description of dark energy, we model the spacetime outside the star with the metric of an exact dark energy solution and summarize the properties of the photon orbits. Moreover, we illustrate the relativistic appearance of a hot spot on a neutron star's surface and study the optical appearance of the surface and the visual size of the neutron star on the screen of an asymptotic observer.

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A Priori Estimates for Solutions of Boundary Value Problems for a Time Fractional Parabolic System of Fractured Porous Media

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A mathematical model of fractured porous media with time memory is considered. It consists of a system of time – fractional parabolic equations for the pressures in each continuum coupled together with an equation of a lower dimension defined on the fractures. By using energy inequalities, we establish a priori estimates for the solutions of boundary value problems corresponding to this system.

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Hundreds of New Satellites of Figure-eight Orbit Computed with High Precision

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In this work we use a Modified Newton's method based on the Continuous analog of Newton's method and high precision arithmetic for a numerical search of new satellites of the famous figure-eight orbit. By making a purposeful search for such satellites, we found over 300 new satellites, including 7 new stable choreographies. Until now there were known only two stable choreographies – the famous Moore's figure-eight orbit and one choreography found by Milovan Shuvakov.

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Machine Learner Ensemble: Statistical Theory and Practice

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Medical decisions often involve the use of longitudinal information collected on individual patients toward predicting likely transitions in their future health status (including death). Accurate predictions help identify patient groups at greatest risk for future adverse events who may be appropriate for targeted intervention. Dynamic prediction models provide predicted survival probabilities that can be updated over time for an individual as new measurements become available. There is substantial interest but limited use of machine learning methods (ML) for dynamic survival prediction. On the other hand, joint modeling and landmarking are popular conventional statistical algorithms for dynamic survival prediction with longitudinal data. The Super Learner is a machine learning ensemble developed by van der Laan *et al* (2007) that combines predictions from different ML and conventional statistical algorithms with the goal of achieving improved performance. It exploits discrete time survival analysis techniques to enable the use of ML algorithms for binary outcomes. Given a bounded loss function, the Super Learner will perform

asymptotically as well as the best individual algorithm and asymptotically as well as the optimal combination of learners. We show how landmarking can be combined with the Super Learner, and discuss practical and theoretical statistical considerations involved in implementing the Super Learner

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Machine Learning

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Machine Learning (ML) is an effective approach for modeling large, complex data, including survival time-to-event data (SD), which are characterized by censoring. Survival curves are routinely generated and their adjustment for specific subgroups requires special care. Estimates of adjusted survival curves can be interpreted as the survival probabilities of groups with similar prognostic covariates. One popular method for analysis of SD is the Cox model, with Proportional Hazards (PH) or the more general stratified model. A standard approach for adjustment of survival curves using results from the Cox model is the means of covariates method, which is often the default in computer survival packages. Here we compared, on left-truncated and right-censored data, several SD-adapted methods (Cox adjusted corrected group prognosis method, Random Survival Forest, Survival Neural Network) with the Cox mean of covariates approach. Con?dence bands for the survival estimates and simultaneous p-values were calculated using a Monte Carlo simulation for pair-wise comparisons over given time periods. The model comparison was carried out on a mid-size real dataset as well as on simulated survival datasets where the simulated ones mimicked the real datasets.

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Spectral Analysis and Long-Time Asymptotics of Complex mKdV Equation

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In this paper, we obtain the long-time asymptotics of the complex mKdV equation via the Defit–Zhou method (nonlinear steepest descent method). The Cauchy problem of the complex mKdV equation is transformed into the corresponding Riemann–Hilbert problem on the basis of the Lax pair and the scattering matrix. After that, Riemann–Hilbert problems are converted through a decomposition of the matrixvalued spectral function and factorizations of the jump matrix for Riemann–Hilbert problem. Finally, by solving the last model problem, the long-time asymptotics of complex mKdV equation are derived.

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Model of Time Series in the Form of a Discrete Analog of the Volterra Integral Equation

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When using the trend analysis methods, it is necessary to build adequate models of time series. Such models are used to test and evaluate the performance of trend analysis algorithms. Well-known models, such as ARIMA, contain a large number of parameters that are difficult to define. Therefore, nonparametric mathematical models of time series have important advantages. The paper proposes a new mathematical model of the time series in the form of a discrete analogue of the Volterra integral equation of the 2nd kind. Such models are the result of applying quadrature formulas and discretization with respect to the arguments of a continuous integral equation. For a separable kernel, analytical solutions of the proposed equations are obtained. In cases where the kernel is a difference one, it is proposed to apply the operational method. An algorithm for identifying the kernel and resolvent from experimental data is considered.

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Hybrid Numerical Experimental Homogenization and Parametric Study Procedure with Application to Porous Media

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This work is devoted to a 3D hybrid numerical-experimental homogenization strategy for determination of elastic characteristics of materials with closed voids. The performed homogenization procedure employs micro-computed tomography (uCT) and instrumented indentation testing data (IIT). Based on the uCT data a 3D geometrical model of a cubic representative elementary volume (RVE) is created assuming periodic microstructure of the material with closed voids. Creating the RVE respects the following principle of equivalence: the porosity assigned to the RVE is the same as the porosity calculated based on the uCT images. Next, this geometrical model is used to generate the respective finite element model where, for simplicity, the voids are considered to have a spherical form. The numerical homogenization technique includes proper periodic boundary conditions with unit force applied in normal and shear directions. The employed solid phase constitutive model is the linear elastic model whose parameters are determined based on IIT data. Accounting the uncertainty in the experimentally determined elastic characteristics and porosity, a parametric study is performed varying the porosity and the elastic modules within the confidence intervals of the corresponding measured values. The obtained results for the characteristics of the homogenized material properties are presented in a graphical form and discussed.

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Numerical Solution of American Option Pricing Problem with Free Boundaries and Volatility Regimes

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In this paper, we consider an alternative model for assessment of American call option with stochastic features of the underlying asset price and volatility transition. The volatility change is expressed as function of macroeconomic and microeconomic conditions as well as unobservable factors. The problem turns to a free boundary problem in a partial differential equation (PDE) system. The volatility of an asset price is not constant and could have a jump when the market conditions are changed. Such a model helps for a more realistic description of the behavior of the asset price and provides a contemporary opportunity for a correct assessment of the American call option value. The model parameters are calibrated and some numerical computations are presented to illustrate the theoretical results. In the problem solution are applied numerical methods. Important remarks for the stochastic features of the asset price are made in the conclusion of the present study.

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Computational Modeling of Carbon Diffusion in bcc Iron in the Presence of 1/2[111] Screw Dislocations

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We present the results of a computational study of a carbon diffusion, trapping and detrapping in the stress field generated by a screw dislocation. To this end, we develop a kinetic Monte-Carlo model, with the behavior of the individual atoms explicitly taken into account. Our model allows to study both the diffusing carbon residing in the dislocation core, and carbon atoms which move through the interstitial sites in dislocation surroundings.

We study the formation of carbon Cottrell atmospheres around screw dislocations at different temperatures and background carbon concentrations by means of long-scale carbon diffusion simulations, making possible to determine both the strength of the carbon atmosphere and the rate of its formation. Within the atomistic kMC model the velocity with which a carbon atmosphere follows a moving dislocation can be estimated, the migration behavior being simulated at experimentally relevant time scales.

We consider the maximal dislocation velocity at which the atmosphere of carbon atoms can follow a moving screw dislocation $v_{\max}(T, c_C)$ as a limit above which screw dislocations break away from the carbon clouds and can not glide slowly and viscously. At lower dislocation velocities, the carbon atoms have sufficient time to follow the dislocation even at lower temperatures, while at higher temperatures, they have sufficiently high mobility to keep up with faster dislocations. The predicted that way velocity range is in agreement with the experimentally observed average velocities of the dislocations gliding via a high-temperature Peierls mechanism.

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Negative Flows of the Soliton Hierarchies

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The soliton hierarchies are characterised by a pair of Lax operators (L and M), L is usually polynomial in the spectral parameter. For the AKNS hierarchy (and its generalisations) for example, L is linear in the spectral parameter. The equations arising from the negative flows (when M involves negative powers of the spectral parameter) are usually equations in non-evolutionary form. These include important examples such as the Camassa-Holm and Qiao equations, when L is in the $sl(2)$ algebra, and the Degasperis-Procesi equation when L is in the $sl(3)$ algebra (with the corresponding reductions). When L is quadratic (the so called quadratic pencil or quadratic bundle) there are negative flows as well, the best known example being the Fokas-Lenells equation (and its extensions on Hermitian symmetric spaces). We discuss other examples, related to the quadratic bundle and the Heisenberg hierarchy, which is gauge equivalent to the AKNS hierarchy.

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The Number of Failed Components in a Failed or Operating Coherent System Consisting of Multiple Types of Components

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We consider a coherent system consisting of multiple types of components. It is of interest to study the number of failed components of each type when the system fails or operates. We extend the results which are well-known in the literature for k-out-of-n systems. The obtained results are next used in the optimization problems.

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Parameter Optimization for Novel Dynamic Vibration Absorbers with Negative Stiffness and Amplifying Component

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Two novel dynamic vibration absorbers(DVA) with negative stiffness and amplifying mechanism are proposed. The dynamic equations are established and it's found that the models designed still have fixed points independent of damping in the amplitude frequency curves of the primary system. The optimum frequency ratio and approximate optimal damping ratio are respectively obtained based on the fixed point theory. To keep the stability of the systems, the optimum negative stiffness ratio and the best working range of inerter are deduced. Furthermore, the influence of different magnification ratio and inerter coefficient on the amplitude-frequency response is analyzed. At last, the control performances of the presented DVAs are compared with three existing typical DVAs, which show that the DVAs presented in this paper have better vibration reduction broadband effect. Moreover, the results of this paper can provide theoretical guidance for the installation position of inerter in the design of vibration absorbers of related structures.

Keywords: Dynamic vibration absorber, negative stiffness, fixed point theory, inerter-mass, Lever component

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Bivariate Compound Poisson Risk Processes with Shocks

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Contemporary insurance theory is concentrated on models with different types of polices and shock events may influence the payments on some of them. Jordanova (2018) considered a model where a shock event contributes to the total claim amount with one and the same value of the claim sizes to different types of polices. Jordanova and Veleva (2021) went a step closer to real life situations and allowed a shock event to cause different claim sizes to different types of polices. In that paper the counting process is assumed to be Multinomial. Here it is replaced with different independent homogeneous Poison processes. The bivariate claim counting process is expressed in two different ways. Its marginals and conditional distributions are totally described. The mean square regression of these processes is computed. The Laplace-Stieltjes transforms and numerical characteristics of the total claim amount processes are obtained. The risk reserve process and the probabilities of ruin in infinite time are discussed. The risk reserve just before the ruin and the deficit (or the severity) at ruin are thoroughly investigated. Their means, probability mass functions and probability generating functions are obtained. Although the model is constructed by a multivariate counting processes, along the paper it is shown that the total claim amount process is stochastically equivalent to a univariate compound Poisson process. These allows us to reduce the considered risk model to a Cramer-Lundberg risk model, to use the corresponding results and to make the conclusions for the new model. Analogous results can be obtained for more types of polices and more types of shock events. The results are applied in case when the claim sizes are exponentially distributed. Explicit formulae for the probability of ruin in infinite time, the distribution of the deficit at ruin and the one of the risk reserve just before the ruin are obtained. Stochastically equivalent models could be analogously constructed in queuing theory.

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Reconstruction of Concentration Sources for Primary and Secondary Pollutant Models

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The complex modeling of the atmospheric pollution process involves a wide range of inverse problems concerned with the identification of physical properties and mass transfer coefficients, internal sources, boundary and/or initial conditions. In this paper, the simultaneous reconstruction of space and time dependent terms in the right-hand side of ultra-parabolic equation describing pollutants emitted from point internal and boundary sources of an urban area is considered. This equation equipped with vertical diffusion coefficient taking zero value on the boundary is an ultra-parabolic equation with degeneration. The direct problem is solved by the splitting method to parabolic and hyperbolic problems. To overcome the degeneration of the diffusion coefficient a fitted finite volume difference method is applied. Then the inverse problem for finding the coefficient functions in the RHS with extra integral overdetermination conditions is considered. Existence and uniqueness of the solution of the inverse problem is discussed. The adequate decomposition of the solution is used for solving the inverse problem. Numerical examples confirming the efficiency of the proposed method are presented.

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Model of the Electromagnetic Field of a Wire Antenna

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The work investigates the electromagnetic field of a wire antenna, such antennas are widely used in practice due to their provision of the necessary characteristics with a relatively simple design. At the same time, a number of questions arise related to the practical application of antennas, namely the influence of the type and design of the antenna on the distribution of current on it, the determination of the input impedance, the standing wave ratio by voltage, the total power, polarization characteristics, etc.

Three main approaches can be distinguished in the study of vibrator antennas: approximate methods using the estimated distribution of the current value on the antenna and methods of superposition of these currents; approximate methods based on replacing the antenna with an equivalent long line; exact methods based on the solution of integro-differential or integral equations in a strict formulation with respect to the unknown value of the complex current on the antenna. In this paper, we consider a mathematical model of the electromagnetic field of a wire antenna, which is an asymmetric vibrator located above the earth's surface; the body of which is the inner wire of the coaxial transmission line.

The purpose of this work is to calculate the electric and magnetic field strengths, analyze the structure of the electromagnetic field of a wire antenna and calculate the field characteristics (input impedance, current distribution). The electromagnetic field of such an antenna in the steady state of radiation is described by the system of Maxwell's equations. On the basis of Maxwell's equations, an analytical representation for the components of the electromagnetic field strengths of an asymmetric antenna is obtained, calculations of the input impedance and current distribution are carried out.

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Higher-order Modified Nonstandard Finite Difference Methods for Autonomous Differential Equations

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Several new classes of nonstandard finite difference methods for solving autonomous differential equations are constructed and analyzed. The new methods are

second-order generalizations of the nonstandard versions of the explicit Euler and Runge-Kutta methods, among others. A set of numerical simulations is presented that supports the theoretical results and highlights the advantages over existing numerical methods.

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Analytical and Computational Analysis of Time Fractional Parabolic Models of of Atmospheric Dispersion of Pollutants

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An example of nontrivial problems in dispersion of pollutants is the description of the diffusion under atmospheric turbulence. A specific consequence of turbulence is the emergence of anomalous diffusion. In recent years, engineers and physicians show that some fractional models give very good results in fitting the experimental data and perform far better than the traditional Gaussian model. In the present work, we investigate the solution behavior of time fractional parabolic advection-diffusion equations modeling atmospheric dispersion of pollutants. We discuss the difficult case of degenerate sub-diffusion, which corresponds to Monin-Obukhov similarity theory of estimating the surface boundary layer with speed and turbulence. We obtain a priory energy estimates for the solution and show well-posedness of the initial boundary problems. Further, using monotone finite difference scheme approximation we find numerically that there should be relation between the order of the fractional derivative with the physical structure of the turbulence flow.

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Finite Time Blow up of Solutions to Klein-Gordon Equations with Variable Coefficient Nonlinearities

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We study the Klein-Gordon equation with two variable coefficient nonlinearities

$$u_{tt} - \Delta u + u = f(x,u), \quad u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \quad t > 0, \quad x \in \mathbb{R}^n,$$

where f has one of the two forms

$$f(x,u) = a(x)|u|^{p-1}u + b(x)|u|^{q-1}u, \quad f(x,u) = a(x)|u|^{p} + b(x)|u|^{q-1}u.$$

We investigate negative, energy subcritical and energy supercritical cases and derive sufficient conditions for finite time blow up of the weak solutions to the Cauchy problem.

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The Applications of POD Method in Rotor-bearing System

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The rotor-bearing system models with misalignment, rub-impact faults and coupling faults are established respectively. The modified proper orthogonal decomposition (POD) – second level order reduction method is proposed via combing the component mode synthesis and POD method. The POD method is used for order reduction to reduce the rotor-bearing system. The efficiency of the POD method is verified via comparing the bifurcation diagrams, amplitude-frequency curves between the original and reduced system models. The vibration behaviors of the reduced rotor system models with single and coupling faults are discussed in details. The results can provide theory guidance to the optimization design of aero-engine.

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Peculiarities of Aerodynamic Loads on a Projectile in Transonic Flight at Small Angles of Attack

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In previous years, a number of numerical and experimental studies showed that aerodynamic characteristics of boat-tail projectiles change intricately at high subsonic speeds of flight. In particular, with increasing free-stream Mach number at a positive angle of attack, the normal aerodynamic force drops rapidly to a so-called critical point before rising sharply to another critical point from which it drops once again. Such abrupt changes in aerodynamic loads during a projectile flight should be obviously taken into account in numerical predictions of the flight stability and trajectory. Meanwhile physical phenomena that cause intricate behavior of the aerodynamic loads were not clearly interpreted.

In the present work, we perform three-dimensional numerical simulations of turbulent airflow over a boat-tail projectile at the angles of attack $\alpha = 4°$ and $\alpha = 6°$ in the range of free-stream Mach numbers $0.938 \leq \mathrm{M}_\infty \leq 0.952$. The simulations are based on the Reynolds-averaged Navier-Stokes partial differential equations. To close the system of equations, we employ a SST $k - \omega$ turbulence model or Detached Eddy Simulation. Numerical solutions are obtained with finite-volume solvers Fluent and CFX of second-order accuracy on fine computational meshes. A detailed analysis of shock wave locations and their interaction with expansion waves and sonic surfaces is performed. Physical interpretations of discontinuous drops and minima of lift & moment coefficients as functions of M_∞ are suggested. We show that the first drop is caused by a coalescence of local supersonic regions on the lower surface of projectile. The second drop and subsequent minimum are explained by a fast expansion of the supersonic region in front of the oblique shock located beneath the projectile. A comparison of the results obtained with Fluent and CFX solvers shows their good agreement.

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Stochastic Resonance of the Spectral Amplification for a Fractional Oscillator in a Fluctuating Magnetic Field

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Motivated by studies of the dynamics of charged particles in plasmas in the presence of a magnetic field, we have considered the dynamics of a charged fractional oscillator with a power-law memory kernel under the action of crossed electric field and a fluctuating magnetic field. The influence of an electric field is considered as a superposition of an external periodic force and an additive stationary noise. Fluctuations of the magnetic field are modeled by the colored dichotomous noise. The main aim of the work is to obtain, in the long-time limit, the exact formulas for the output first-order statistical moments generated by the model considered. The Shapiro-Loginov formula with the Laplace transformation technique allows us to achieve this objective. A major virtue of the investigated model is that on interplay of a noisy magnetic field and the external periodic forcing in a charged, fractional oscillator can generate a rich variety of nonequilibrium cooperation effects. As one of the main results we have established the non-monotonic dependence of the amplitude of the mean oscillator displacement on the parameters of the magnetic field fluctuations (stochastic resonance). As another main result we have found the effect of a very sensitive response of the mean oscillator displacement to small variations of the dichotomous noise amplitude at small values of the memory exponent (strong memory), i.e., the spectral amplification displays a quick jump from a very high value to a low one as the dichotomous noise amplitude increases but a little. We believe that the results obtained are of interest in the field of plasmas and also in cell-biology, where issues of memory and multiplicative colored noise can be crucial.

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Influence of a Fluctuating Magnetic Field on the Statistical Moments of a Charged Oscillator Driven by a Periodic Force

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Diffusion of particles in plasmas exposed to an external magnetic field still remains one of the important problems of plasma physics and controlled fusion. Inspired by studies in this field, we have considered the long-time behavior of the second-order statistical moments for a charged Brownian particle in a harmonic potential well, which is simultaneously exposed to a fluctuating magnetic field. Fluctuations of a magnetic field are modeled by the white noise. Moreover, to make the model more general, we add an external electric force, which is expressed as a sum of two contributions: a stationary colored noise and a periodic electric field. The main contribution of this work is as follows. In the long-time limit, we provide exact formulas for the analytic treatment of the dependence of second-order statistical moments of oscillators displacements, such as variances of fluctuating position and velocity of the Brownian particle, on system parameters. Also, the cross-correlation between particle fluctuations along orthogonal directions as well as the mean angular momentum are found. As our main result, we have established that in presence of magnetic field fluctuations the variances of particles' position and velocity depend on the parameters of external perioding forcing. Particularly, it is shown that an interplay of the fluctuations of the magnetic field and external periodic driving induce a cross-correlation between particler's velocity components. These effects are somewhat surprising since in a constant magnetic field such a cross-correlation is absent and the variances are independent of external periodic forcing.

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Reaction Networks with Application to Epidemiological Outbreaks and Biological Growth

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Reaction networks are useful tools for modeling and simulation of epidemiological outbreaks and biological growth processes (see [1-4]). Several classes of reaction networks possessing clear physical or chemical interpretation together with the induced dynamical systems are discussed and analyzed. The studied reaction networks include basic reactions such as exponential radioactive decay, logistic and Gompertz growth. We mathematically analyze the solutions to the generated systems of ordinary differential equations, as well as the absolute and relative (logarithmic) rates of change of these solutions. We then graphically present the obtained solutions and their rates for different parameterizations and look for typical characteristics of the shapes in the graphs, such as inflection points and lag times (if any), behavior near asymptotes, etc. We also check the fit of the solution functions obtained against different datasets to draw conclusions about the domain of applicability of the various reaction networks.

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Computational Approach to the Inhibition of SARS-CoV-2 Orf6

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In addition to its 4 structural and 16 non-structural proteins, the SARS-CoV-2 virus expresses also 9 accessory proteins that are not necessary for in vivo replication but are responsible for the virus's pathogenicity and regulating innate immunity. One of the most important SARS-CoV-2 accessory proteins is the Orf6.

The SARS-CoV-2 Orf6 is a small protein, containing 61 amino acid residues. Its primary function is to antagonize with type-I interferon signaling by targeting the Rae1-Nup98 complex, which contributes to mRNA nuclear export. This is how the host innate immune antiviral response is supressed. Orf6 is a membrane protein but contains a highly acidic and flexible C-terminal part, which is considered responsible for the interaction of the protein with the cellular Rae1-Nup98 complex. It was shown that the SARS-CoV-2 Orf6 contributes significantly to COVID-19 lung pathology and disease outcome. This necessitates the search for possible inhibitors of this protein.

Interferon gamma (hIFNγ) is a key signalling molecule, playing an essential role in the initiation and modulation of the immune response in the body. It is a homodimer, organized as an α-helical globule and two highly flexible C-terminal tails. With its basic sequence, this particular domain of IFNg is a perfect binding partner candidate for neutralizing the acidic SARS-CoV-2 Orf6 C-terminus.

Here we report our molecular modeling studies on the possible inhibition of the SARS-CoV-2 Orf6 by either a full-length hIFNγ homodimer, or a few peptides, encompassing the C-terminal domain of the cytokine. Our *in silico* experiments demonstrate that the two molecules interact with high affinity due to the strong electrostatic attraction between them. Upon binding of either hIFNγ C-terminal peptides or the full-length cytokine molecule, the C-terminal domain of the SARS-CoV-2 Orf6 is effectively neutralized, which would potentially impede its binding to the Rae1-Nup98 complex.

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BURA Preconditioning of Multiphysics Problems

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Multiphysics or multiscale problems naturally involve coupling at interfaces, which are manifolds of lower dimensions, thus giving rise to interface conditions formulated in fractional Sobolev spaces. The block-diagonal preconditioning of the related saddle-point systems is among the most efficient approaches for solution of large-scale problems of this class. The implementation of such preconditioners requires a proper approximation of the inverse of the discrete fractional Laplacian at the interface. we propose to use the best uniform rational approximation (BURA) method for preconditioning of such problems. The presented analysis is focused on the convergence properties of the preconditioned iterative solution method. The estimates d not depend on the smoothness properties of the solution. Numerical tests well illustrate the convergence properties of the preconditioned iterative method.

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Bifurcations and Vibration Analysis of a Four-Dimensional Chaotic Hyperjerk Circuit System

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In this paper, the bifurcations and nonlinear vibrations of a hyperjerk circuit system are investigated via the analytical methods and numerical calculations. Based on the high-dimensional bifurcation theory and perturbation method, the analytical criteria and properties of Hopf bifurcation are discussed. We further analyze the stability and analytical approximate expressions of periodic orbits via the harmonic balance method and Floquet theory. The critical conditions of flip bifurcation, fold bifurcation and symmetry-breaking bifurcation are explored in detail. The accurate positions of bifurcations are also obtained. A semi-analytical and semi-numerical method is proposed to explore two routes to chaos. We also analyze abundant nonlinear vibrations, including multi-stability, anti-monotonicity and boundary crisis. It is found that multiple Feigenbaum trees coexist in parallel, which have two bifurcation modes. In a certain parameter space and time interval, the region is estimated analytically where chaotic dynamics cannot emerge.

The physical circuits and synchronization control schemes of two chaotic hyperjerk systems are designed. To establish the accurate parameter relationships at bifurcation points, we apply the theoretical methods of periodic solution and bifurcation to the proposed electronic circuit model. It can be used to identify the type of bifurcations appearing in system quickly, so as to provide an important theoretical reference for the analysis of dynamical characteristics, bifurcation control and fault diagnosis of circuits efficiently.

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Presentation of Data for Solving Problems of Artificial Intelligence

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Use of artificial intelligence is becoming a topical issue. The development of intelligent systems related to various tasks where are the main of which is the preservation and presentation of information. That is the way the data base was inspired and the necessary information is taken away by the way. One method of using a relational data base and using machine learning methods for searching, which allows the system to learn for more personalized responses to requests. Another idea for using non-relational NoSQL databases for the implementation of artificial intelligence systems. We apply two approaches. The module for searching scientific information for this purpose will be presented in accordance with the use of different types of databases, given that any complex behavior is a large set of conditioned reactions to stimuli. Data about the number of causes for the formation of reactions, it is necessary to save. Formed recommendations for the user, as presented by the translation of scientific articles, which are formed on the basis of the request. The robotic algorithm is composed of such points. When entering the system, the requests are saved in the key-value system, becoming immediately available for the recommendation system. We gave a recommendation system based on the indications in a similar way to start sorting out nodes. Having taken a lot of knots, the system re-verifies them for similarity. Whether a given extract is representative instead text keywords. When offending, they realize the task, but in the case of the skin, they have their own advantages and shortcomings.

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The Multi-Peakon Weak Solution for the Rotation-Two-Component Camassa-Holm System

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This talk is concerned with the rotation-two-component Camassa-Holm (R2CH) system, which is a model for the equatorial water waves with influence of the Coriolis force. We study whether the R2CH system admits multi-peakon weak solutions in distribution sense. Firstly, we construct a solution of the multi-peakon form and substitute this solution into the traveling wave transformed ordinary differential equation. Then, it can be found the two-peakon solutions of this form exists. Finally, We discuss some of the solutions in different cases and draw the phase diagrams.

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Numerical Modeling of the Noise Generated by Hydraulic Systems with Centrifugal Fan at Two Flow Rate Adjustment Methods

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The paper presents the results from both experimental tests of noise generated by a hydraulic system with a centrifugal fan and statistical analysis. Adjustment was performed by applying Variable-frequency drive at the supplying electromotor and by using inlet guide vanes. The noise analyzes were performed, as a function of the adjustment method. The parameters that were measured include - A-weighted sound pressure level and sound pressure level in octave bands. The noise values for each second were measured and this was done for 120 values for each operating mode of the system. This makes a database of 2280 observations and includes variables as rotational speed of the fan shaft, angle of the inlet vanes, flow rate, octave bands of the noise, and A-weighted sound pressure level.

The normal distribution law was tested on the applicable data. A correlation and factor analysis were performed in order to study the relationship between the variables. Different regression models were obtained to study the relationship

between operating modes of the hydraulic system and the levels of emitted noise. A comparison analysis of the results was done.

As fan systems are widely used in different types of manufacture (in both industrial and residential buildings), it is important to define the link between noise and operation and to preserve the health of the humans affected.

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Statistic Study of Particulate Matter (PM10) Air Contamination in the City of Svishtov, Bulgaria

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The high levels of all air pollutants are of great importance for the human health. Especially dangerous are particulate matter (PM) contaminants. Usually PM is a mixture of solid particles and liquid droplets found in the air. Particle pollution includes: PM10: inhalable particles, with diameters that are between 2.5 and 10 micrometers; these particles are emitted directly from a source, such as construction sites, unpaved roads, fields, smokestacks or fires. Most particles form in the atmosphere as a result of complex reactions of chemicals such as sulfur dioxide and nitrogen oxides, which are pollutants emitted from power plants, industries and automobiles. For all European countries there are many regulations aimed against the air contamination and for the monitoring of pollution. Bulgaria, as a part of EU has to follow all these regulations. Still in Bulgaria there are many places and periods of time during the years with PM10 contamination, bigger than the daily norm of $50\boldsymbol{m}\mathrm{g/m}^3$. This study is to investigate the PM10 air pollution in the city of Svishtov, Bulgaria for the period 2012–2022. The town of Svishtov is located in northern Bulgaria, on the banks of the river of Danube — the north Bulgarian border with Romania. For the study we use official data from the monitoring of PM10 in Svishtov region, Bulgaria. We apply different statistic methods to study data and to predict future PM10 pollution in the city of Svishtov. All result of study are graphically presented and commented.

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Coupled-Cluster Calculation of Neutron Matter Equation of State

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We present a relation between the density and pressure of pure neutron matter derived from numerical calculations based on a truncated coupled cluster model using a simple bare nucleon-nucleon interaction in the Minnesota potential form. The obtained relationship exhibits well-known behavior only at low densities $\lesssim 0.08\text{fm}^{-3}$ and diverges from the supported by observations data equations of state at higher densities. Our results are compared with those derived from more realistic potentials from the effective field theory using realistic NN forces.

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A Numerical Study of the Effect of Thermal Relaxation on the Photothermal Signal from Semiconductors

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Photothermal techniques are extremely useful as non-contact and non-destructive methods for material characterization. All photothermal techniques share a common feature which is the modulated excitation of the material, usually using an intensity modulated light beam which is partly absorbed in the material. The mathematical treatment of the subsequent heat diffusion in the material assuming Fourier's heat transport theory, gives rise to thermal excitations that resemble wave like solutions known as thermal waves. The detection of thermal waves with various ways can reveal the thermal transport properties of the materials. Lately, the introduction of thermal relaxation (Cattaneo heat transport) has attracted much attention, giving the possibility for more accurate characterization in materials with thermal memory. In this work, we investigate the photothermal signal from a semiconductor which includes an additional component due to the excitation of electron-hole pairs (free carriers). The free carrier diffusion results in a wave like solution similar to the thermal wave solution. The excursion in the free carrier

concentration serves as an additional heat source and contributes to the net photothermal radiometric signal.

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Numerical Method for Determining the Instantaneous Flow Rate of a Three-Rotor Gear Pump with Bilateral Lantern Meshing

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The present work is a continuation of the research of a new type of pump with bilateral lantern meshing. So far, the dependencies for determining the volume of the working chambers of the machine as a function of the angle of rotation of its shaft and the independent geometric parameters of the machine have been found: scale module – m; number of tubes (teeth) of the tubular wheel – z; epi- and hypocycloid shortening coefficient – λ; coefficient of the radius of the forming circle (barrel).

In this work, a numerical method for determining the instantaneous volumetric flow rate and the non-uniformity of the flow rate of a three-rotor gear pump with bilateral lantern meshing is developed. For this purpose, the dependencies are determined and an algorithm for calculating these variables is presented. The instantaneous flow rate is calculated as a function of the angle of rotation of the pump. Using this method, a calculation module using VBA and Excel is created and it is presented a numerical example of a pump with an odd number of pipes. This method will make it possible in the future to investigate the nature of the change in flow rate and how the geometric parameters of the machine affect the non-uniformity of this flow rate.

Keywords: Gear pump, lantern meshing, hypocycloid meshing, epicycloid meshing, instantaneous volumetric flow rate, numerical method, coefficient of the non-uniformity of the flow rate

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Review of Inductive Magnetic Measurement Techniques: AC Susceptibility and Torque Cantilever Magnetometry: Theory and Experiment

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I will review two important magnetic measurement methods: ac susceptibility and magnetic torque cantilever magnetometry. Theoretical foundation (using fundamental physics equations) and experimental design for measurements in various ac and dc magnetic fields will be discussed. I will show various data sets ranging from superconductors to Kondo insulators to show the usefulness and suitability of both methods to learn new magnetic properties in various materials.

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Inference for a Multilevel Step-Stress Model with Censored Exponential Data

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In many industrial life-testing experiments, the units under investigation are extremely reliable and have large mean times to failure under normal operating conditions (NOC). Step-stress is a special type of accelerated life-testing procedure that allows the experimenter to test the units of interest under various stress conditions changed (usually increased) at different intermediate time points. In this talk, we consider a stochastic step-stress model with exponential lifetimes and under Type-II censoring. We further assume that the impact of the experienced stress level change follows the cumulative exposure model under which the lifetime of a single unit has an absolute continuous cumulative distribution function. The main interest of the study is on inference for the mean lifetime (failure rate) at each stress level. We apply several statistical techniques for estimating the unknown underlying parameters. The hypothesis testing problem about the scale parameters is considered under a log-link connection between the stress and the mean failure times. The associated likelihood-ratio statistic is modified by making use of the independent and scale invariant properties of the normalized exponential spacings.

Furthermore, we extrapolate the presented results to NOC by using the assumed life-stress relationship.

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Statistic Study of Gaseous Air Contamination in the City of Ruse, Bulgaria

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It is well known that the high levels of all air pollutants are of great importance for the human health. Especially dangerous are gaseous and particulate matter contaminants. The gaseous air pollutants of primary concern in urban settings include sulphur dioxide, nitrogen dioxide, and carbon monoxide. They are emitted directly into the air from fossil fuels such as fuel oil, gasoline, and natural gas that are burned in power plants, automobiles, and other combustion sources. Ozone (a key component of smog) is also a gaseous which is formed in the atmosphere via complex chemical reactions occurring between nitrogen dioxide and various volatile organic compounds as gasoline vapors. There are many regulations aimed against the air contamination and for the monitoring of pollution for all European countries. Bulgaria, as a part of EU has to follow all these regulations. Still in Bulgaria there are many places and periods of time during the years with air contamination, bigger than the daily norms. This study is devoted to examine air pollution with gaseous substances in the Ruse region, Bulgaria. The city of Ruse is located in northeast Bulgaria, on the banks of the river Danube – the north Bulgarian border with Romania. It presents a statistical analysis of the level of air pollution in Ruse on official data from the monitoring stations in the city. The measurements cover the period 2015-2022. All results of the study are graphically presented and commented.

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Mathematical Modeling of the Electrochemical Phase Formation through a Supercooled Liquid State Stage in Metals

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The existence of the phenomenon of the electrochemical phase formation in metals via a supercooled liquid state stage is discussed. Emergence and ultrarapid solidification of a deeply supercooled metallic liquid represented by a multitude of liquid atomic clusters, ensures phase transformations at the interface. A mathematical model has been obtained describing the liquid state behavior of metals being electrodeposited and their subsequent transition to the solid state. The model results confirm the existence of the phenomenon under discussion.

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Sensitivity Analysis in Air Pollution Modeling by Innovative Monte Carlo Methods

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In this paper a systematic procedure for multidimensional sensitivity analysis of a case study in the area of air pollution modeling has been done. The Unified Danish Eulerian Model (UNI-DEM) is used in our investigation as one of the most advanced large-scale mathematical models that describes adequately all physical and chemical processes. Sensitivity study of the output of UNI-DEM according to emission levels and chemical reaction rates is performed. Sensitivity Analysis decomposes the uncertainty in inference to uncertainty in inputs to identify which inputs are relevant for the prediction and then investigate how their uncertainty can be reduced in order to improve the accuracy of the prediction. We discuss a systematic approach for sensitivity analysis studies in the area of air pollution modeling. Different parts of the large amount of output data, produced by the model, were used in various practical applications, where the reliability of this data should be properly estimated. Another reason to choose this model as a case study here is its sophisticated chemical scheme, where all relevant chemical processes in the atmosphere are accurately represented. We study the sensitivity of concentration variations of some of the most dangerous air pollutants with respect

to the anthropogenic emissions levels and with respect to some chemical reactions rates. A comprehensive experimental study of Monte Carlo algorithm based on randomized, center and edge latin hypercube sampling for multidimensional numerical integration has been done. Samplings with different seeds has been analyzed. We use a division of the distribution of each variable into equiprobable intervals. The values obtained for each variable are paired randomly. This has been made for the first time for sensitivity analysis of UNI-DEM. The algorithms have been successfully applied to compute global Sobol sensitivity measures corresponding to the six chemical reactions rates and four different groups of pollutants.

Acknowledgement. This study is supported by the Bulgarian National Science Fund under Project KP-06-M32/2–17.12.2019 "Advanced Stochastic and Deterministic Approaches for Large-Scale Problems of Computational Mathematics" and Project KP-06-N52/5 "Efficient methods for modeling, optimization and decision making."

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Systems Subject to Marshall-Olkin Type Shocks

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This study investigates a two-component series system using the Marshall-Olkin extreme shock model and the Marshall-Olkin run shock model (see [1-7]). According to the Marshall-Olkin extreme shock model, fatal shocks whose magnitude is greater than or equal to a predefined threshold value d produced from source i affect the component i, on the other hand fatal shocks produced from the source 3 affect both components simultaneously, $i = 1, 2$. In the Marshall-Olkin run shock model, on the other hand, there should be k successive fatal shocks from the same source. When the interarrival times between shocks follow a phase-type distribution, the system's reliability, mean residual lifetime, and mean time to failure are analyzed in detail.

Key Words: Marshall-Olkin distribution, reliability, mean residual lifetime, phase-type distributions, mean time to failure

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On the Transferability of the Deep Galerkin Method (DGM) for Solving Partial Differential Equations

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Recently, there has been an increase in the development and use of machine learning (ML) techniques for the solution of partial differential equations (PDEs). Popular methods are Physics Informed Neural Networks (PINN), Deep Galerkin Method (DGM) and the Deep Ritz Method (DRM) amongst others. The assessment and improvement of the performance of these methods on different types of problems, is an active research topic.

In the current work we investigate the transfer of knowledge in the context of the DGM algorithm. Specifically, we examine how well a DGM network pre-trained on one type of PDE problem, performs on related problems. This has the potential of enhancing the applicability of the method and reducing the training time of the neural networks involved.

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Some Combinatorial Principles and Their Applications to C^* Algebras

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As is stated in [1] the strong form of the Jensen Diamond provides a $\aleph_1$-generated counterexample of the famous Naimark's problem, i.e, the C^* algebra of the compact operators over some Hilbert space $\mathcal{K}(H)$ is the only one with the following property: "Let ρ_i, $p = 1, 2$ be any two irreducible representations, so they are unitarily equivalent (homotopic) to the trivial representation."

By constructing an example of a topological recursion, assuming the Diamand [1] manage to create a nice counterexample. We will investigate (continuing the work in [2,3]) some of the consequences of the aforementioned and quite similar combinatorial principles. For instance, of main interest is the following: Are there some consequences of NCF principle to the calculation of the analytical index of Fredholm operators by using the Atiah sequences. As well, what about the dependence of the set theoretic methods and K theory. For what does it mean NCF, see, i.e, [2,3].

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On the Theory of the Fredholm Operators That Arise Naturally from the Generalized Timoshenko Beam System with Several Boundary Conditions

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In [1] we studied the sufficient (which actually are necessary as well) to be the corresponding Cantilever Timoshenko System stable. The reason of being admitted “nice"solutions, however is that the system is indeed conservative and the corresponding local energy density can be treated as a norm of a Classical Sobolev Space, where the operator of the system results to be generic of a semigroup of a continuous contractions.

Actually, the theory of semigroups is widely studied and applied in Bresse Systems, in particular, in establishing the rate of decay of a stable solutions – we can use the spectral properties provided by the corresponding Semigroups of Bounded operators. The type of the Semigroup and moreover the spectral properties in several cases relies on the fact that these problems are equivalent to each other modulus Compact operators and, in particular, have the same essential spectrum. It is studied the connection between the main problem with the Fredholm operators mentioned above and it is given and solved a prticular example generalizing [1].

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The Initial Value Problem of the Gelfand-Dickey Hierarchy

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In this talk I will summarize the solution to the smooth initial value problem of the (commutative or non-commutative) Gelfand-Dickey (GD) hierarchy. A crucial problem is to decide how to interpret the word "smooth" in this case, since the GD hierarchy is defined in terms of formal pseudo-differential operators. I will show how this problem can be solved by using some recent approaches to differentiability in infinite dimensions that do not use the notion of an atlas.

This work is collaboration with J.P. Magnot and V. Roubtsov.

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On Nonstrictly Hyperbolic Systems, Their Regularizations, and Models of Natural Sciences Reducible to Them

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We show that when modeling some phenomena of various nature, for which oscillatory processes are natural, nonstrictly hyperbolic systems of nonlinear equations of a special form or their parabolic regularizations arise. Such systems can be studied in a similar way. As examples, we give a model of electron plasma, a model of a stratified fluid, and a model of movement of a fluid in a closed tube with reacting walls (a prototype of the blood circulatory system). In all these cases, the oscillations occur naturally, they can be regular or blow up with time. In addition, traveling waves can exist in all these systems.

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Stochastic Modeling of Tree Growth Regulatory Networks

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Tree growth regulation networks are dynamic and stochastic, with feedback that regulates their growth function at different ages. Modeling of the tree growth process poses new challenges due to the large number of trees in the stand and the stochastic nature of their interactions. In this study, we motivate stochastic modeling of tree growth networks and demonstrate the method using a 4-parameter Gompertz-type diffusion process and an observed data set from permanent sample plots in Lithuania. In order to cover a wider range of forest stands, random effects are additionally included in the model. The newly developed mathematical model is calibrated by determining the numerical values of the parameters in order to best respond to the measurements collected during the experiment.

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Multiple Periodic Vibrations of Auxetic Honeycomb Sandwich Plates Subjected to In-Plane and Transverse Excitation

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Sandwich plates in auxetic honeycombs with special properties can meet the needs of modern science and technology development and have been widely used. A major concern in the design and construction stages is to predict the force and nonlinear vibration behaviors. To solve this problem, the Melnikov method is extended to the study of multiple periodic vibrations of auxetic honeycomb sandwich plates subjected to in-plane and transverse excitation. The effects of

transverse excitation on nonlinear vibration behaviors are discussed in detail. Evolution laws and waveforms of multiple periodic vibrations are obtained to analyze the energy transfer process between the first two order modes. The changes of number and amplitude of the multiple periodic orbits are intuitive described in the phase-parameter bifurcation portrait.

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Minimax Estimation of the Forecast Set of the Dynamical Economic and Mathematical Model of the Production System

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The article is devoted to the application of discrete-time controlled dynamical economic and mathematical models of production systems for solving the problem of constructing and minimax estimation of its final predictive set (the final reachability set of the system). On a given integer time interval, an economic and mathematical model of a controlled production system (controlled dynamical system) is considered, which is described by a system of linear discrete-time recurrent equations. In each period of time, the values of the control action are bounded by the corresponding nonempty convex compact polyhedron, and the values of the phase vector of the dynamical system are bounded by the corresponding finite joint system of linear algebraic equations and inequalities. In the article describes the formalization of the problem of minimax estimation of the final predictive set of phase vectors of the dynamical system under consideration, as well as the proposed algorithm for its solution. The results obtained in the article can be used to constructing intelligent computer systems for information support in solving actual technical, economic and medical problems.

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Dynamical Behaviors of Nonlinear Wave Equation in Inhomogeneous Media

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In this talk, we consider the nonlinear wave equation in inhomogeneous media, which is modeled by the variable coefficient wave equation. By taking the periodic solutions as a breakthrough, we study the dynamical complexity of its solutions and find the essentially different dynamical behaviors for the spatially inhomogeneous wave equation.

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Contact Pressure over Doubly Connected Rectangular Domains and Punch Shape Optimization

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Contact problems arises in a variety of industrial processes, engineering and biomechanical systems. Each machine is a set of interacting parts, and necessary to ensure their contact strength and rigidity exists. 3-D contact problem for a rigid punch with a doubly connected base bounded by the lines close to rectangles is in consideration. An analytic-numerical technique is developed for its solving. The problem contains Fredholm integral equations of the first kind, which are treated into the second kind by means of regularization. Using the simple layer potential expansion, the kernels of the integrals are presented in the form of expansions in the powers of the polar radius. The difference between the values of the desired function at different points and the subsequent interpolation of the terms are proposed to smooth the kernels and eliminate singularities. The integral equations are reduced to one-dimensional and then solved using quadrature formulas. Then a punch shape is taken as a desired function, and the role of a minimizing functional is played by the root-mean-square deviation of the pressure distribution arising under the punch from some optimal distribution. In this case, the values of the total forces and moments applied to the punch are assumed to be given, which leads to restrictions imposed on the distributions by the equilibrium conditions. The normal displacements are determined which arising under the action of the found contact pressure on the elastic half-space. The desired punch shape is found using the simple layer potential. A solution to the problem is obtained for the punch with the doubly connected base bounded by lines close to rectangles.

Acknowledgements. This research was supported by CRC TRR154 grant.

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Analysis of Nonlinear Weakly Singular Integro-Partial Differential Equations Arising from Viscoelasticity

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An efficient matrix method is presented for the solution of non-linear weakly singular partial integro-differential equation (SPIDE) arising from viscoelasticity subject to the given initial and boundary conditions. The method is based on the operational matrices of Legendre wavelets. By implementing the operational matrices of Legendre wavelets, the given integro-PDE is reduced to the system of nonlinear equations. Some useful results concerning the convergence and error estimates associated to the suggested scheme are presented. Illustrative examples are provided to show the effectiveness and accuracy of proposed numerical method.

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Machine Learning Based Parameter Estimation of Multitype Branching Processes

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We consider two-type branching stochastic processes with offspring distributions from the power series distribution family. The estimation of the individual parameters is an important part of the statistical inference for branching processes. Models of multitype branching processes require a large amount of data that cannot always be observed. The size of the population is often insufficient. The use of approximation

methods like the machine learning approach to parameter estimation, allows us to obtain an algorithmic estimation in the presence of hidden data. Examples of two-type branching processes are provided, as well as a software implementation, illustrated by simulations and computational results in Python.

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Data Transformations and Transfer Functions

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During the talk we speak on learning mechanisms of data transformation and aggregation. We will introduce SPOCU transfer function and provide some of its unique properties for processing of complex data [1]. Statistical learning will be also discussed.

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Optimization of a Novel Viscoelastic Dynamic Vibration Absorber with Lever, Inerter and Negative Stiffness

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Vibration under complex working conditions is an important factor affecting the efficiency, reliability and safety of equipment structure. Lever, inerter and negative stiffness have good vibration control performance. In this paper, Maxwell model with viscoelastic characteristics is introduced into dynamic vibration absorber, a novel DVA model with lever, inerter and negative stiffness is proposed. There are three fixed points in the amplitude-frequency curve of the system that are independent of damping. Based on the fixed-point theory and optimization criterion, we obtain the optimal frequency ratio and the optimal damping ratio, the optimal negative stiffness ratio is selected and the optimal working range of inerter is calculated under the premise of ensuring the stability of the system. In addition, we analysis the effects of lever amplification ratio and inerter coefficient on the amplitude of the primary system. Finally, compared with the classical DVAs, the performance of the novel DVA is better under harmonic excitation and random excitation, which provides a theoretical basis for the design of a novel viscoelastic DVA.

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Bifurcation and Number of Subharmonic Solutions of Some $2n$-Dimensional Systems and Its Application

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In high dimension, the bifurcation theory of periodic orbits of nonlinear dynamics systems are difficult to establish in general. We consider the existence and bifurcation

of subharmonic solutions of two types of 2n-dimensional nonlinear systems with time-dependent perturbations and time-independent perturbations respectively.

When the unperturbed system is a Hamiltonian system, we study the extended Melnikov function by means of performing the curvilinear coordinate frame and constructing a Poincaré map. According to different disturbance forms, we establish different curvilinear coordinate frames. Then we study the number of subharmonic solutions bifurcate from different invariant torus or different locations of one invariant torus. The results obtained in this paper contain and improve the existing results for $n = 2, 3$.

When the unperturbed system contains an isolated invariant torus, we use the same method as above to study the condition for a singular invariant torus to bifurcate subharmonic solutions under different disturbance forms. All of the subharmonic solutions bifurcate from the singular invariant torus. We study the number of subharmonic solutions bifurcate from this singular invariant torus.

We apply the extended Melnikov method to study the bifurcation and number of subharmonic solutions of the ice covered suspension system. The maximum number of subharmonic solutions of this system is 2 and the relative parameter control condition is obtained.

Keywords: $2n$-dimensional nonlinear system, bifurcation, curvilinear coordinate, subharmonic solutions

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Multidimensional Sensitivity Analysis in Air Pollution Modeling by Lattice Rules with Optimal Generating Vectors

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Recently, there has been an increase in the development and use of machine learning (ML) techniques for the solution of partial differential equations (PDEs). Popular methods are Physics Informed Neural Networks (PINN), Deep Galerkin Method (DGM) and the Deep Ritz Method (DRM) amongst others. The assessment and improvement of the performance of these methods on different types of problems, is an active research topic.

In the current work we investigate the transfer of knowledge in the context of the DGM algorithm. Specifically, we examine how well a DGM network pre-trained on one type of PDE problem, performs on related problems. This has the potential of enhancing the applicability of the method and reducing the training time of the neural networks involved.

An important issue when large-scale mathematical models are used to support decision makers is their reliability. Sensitivity analysis of model outputs to variation or natural uncertainties of model inputs is very significant for improving the reliability of these models. By definition sensitivity analysis is a procedure for studying how sensitive are the output results of large-scale mathematical models to some uncertainties of the input data. The computational efficiency (in terms of relative error and computational time) of advanced Monte Carlo algorithms for multidimensional numerical integration has been studied to analyze the sensitivity of UNI-DEM model output to variation of input emissions of the anthropogenic pollutants and of rates of several chemical reactions. The algorithms will be applied to compute global Sobol sensitivity measures corresponding to the influence of several input parameters on the concentrations of important air pollutants. The study will be done for the areas of several European cities with different geographical locations.

For the first time different modifications of lattice rules with different optimal generating vectors will be applied for the problem under consideration. At the first step of our algorithm the optimal vectors will be generated by the recently developed component by component fast construction method. The second step of the algorithm is generating the points of lattice rule by formula in the definition of the one ranked lattice rule. And at the third step of the algorithm an approximate value of the multidimensional integral is evaluated by the Monte Carlo approximation formula. The obtained lattice rule with our optimal generating vector has an optimal rate of convergences for the corresponding class of functions which define the sensitivity indices in the multidimensional air pollution model. The developed lattice methods based on optimal generating vectors will be compared with the best available stochastic approaches for measuring the total sensitivity indices.

Acknowledgement. This study is supported by the Bulgarian National Science Fund under Project KP-06-M32/2–17.12.2019 “Advanced Stochastic and Deterministic Approaches for Large-Scale Problems of Computational Mathematics” and Project KP-06-N52/5 “Efficient methods for modeling, optimization and decision making.”

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Soliton Interactions for the Boussinesq Equation

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In this paper, we discuss a general decomposition for solutions obtained through the Hirota substitution method from linear combinations of exponents. In particular, we study the Wronskian solutions for the good Boussinesq (gB) equation. The asymptotical analysis is carried in terms of multi-linear functions. We show that a multi-soliton solution for the good Boussinesq equation, which involves interaction of two resonant solitons, always develops singularity.

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Machine Learning Approach to Study and Predict Air Pollution

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Air pollution is a modern problem of high importance due to its negative impact on human health. The Organization for Economic Co-operation and Development (OECD) predicts that in 2050, air pollution will be the leading cause of environmental deaths worldwide. In addition, air pollution is classified as a major cause of cancer, related to the environment. In recent years, machine learning techniques have been used extensively to predict future values of time series. They can model nonlinear dependencies, allow the inclusion of a large number of features and have weaker requirements than those of classical statistical methods. Fine particulate matter (PM) is the most common and major air pollutant in the city of

Ruse, Bulgaria and is a serious problem for air quality. The paper uses the xgboost method to predict the air pollutant PM10. Data for the city of Ruse for the period 2015–2021 were used. The model includes meteorological indicators such as: cloud cover, wind strength and direction, temperature, precipitation, air pressure, etc. The calculations were performed with the Python programming language.

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A Symplectic High-Order Accurate Numerical Method for the Sixth Order Boussinesq Equation

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In this talk the one-dimensional Boussinesq equation with a sixth order space derivative is considered. The numerical method used for its solution is constructed after the representation of the equation as a Hamiltonian system. The space derivatives are replaced with fourth order of approximation finite differences, which leads to a semi-discrete finite-dimensional Hamiltonian system. For the time discretization we apply a symplectic partitioned Runge-Kutta method with 3-stage Lobatto IIIA and IIIB coefficients. The numerical solution preserves the symplectic structure on the discrete level. Numerical experiments are provided for two specific problems with quadratic and cubic nonlinearities, i.e., for propagation of a single solitary wave and interaction of two waves traveling toward each other. The numerical results show $O(h^4 + \tau^4)$ order of convergence of the discrete solution to the exact one.

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An Algorithm for Melnikov Functions and Application to a Chaotic Rotor

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This talk is concerning a chaotic oscillation of a nonlinear model of turbine rotor. Melnikov's method is one of the most important methods in determining chaos, but the practical model prevents the application of Melnikov's method because its complicated nonlinearity makes difficulties to count the number of equilibria, not mentioning the determination of stability and the computation of heteroclinic orbits. In this work, a numerical algorithm is given to compute Melnikov functions with an idea of avoiding the computation of heteroclinic orbits. The convergence of the algorithm and the estimates of errors help us to give a parameter region for chaotic oscillation of the rotor.

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Vibration Suppression of Nonlinear Composite Laminated Plate Using Nonlinear Energy Sink with Internal Nonlinear Oscillator

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Nonlinear energy sink (NES) has great advantages in suppressing harmful vibration of composite structures. However, once the excitation amplitude becomes larger, the vibration suppression performance of NES will be greatly reduced or even ineffective. To solve this problem, a grounded nonlinear energy sink with internal nonlinear oscillator is proposed and studied in this paper. Based on the first-order shear deformation theory and Galerkin method, the dynamic model of nonlinear composite laminated plate with grounded NES with internal nonlinear oscillator is established. First, the vibration reduction superiority of the grounded NES compared with traditional NES under shock excitation is analyzed by numerical method. Then, the slow flow equations of the system are derived by using complexification averaging method, and the validity of the derivation is verified. By comparing the frequency response under different excitation amplitudes, the great advantage of the grounded NES in suppressing the emergence of high branches is analyzed.

Finally, the cooperation of parameters between different oscillators is investigated to further improve the vibration reduction performance of the grounded NES.

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Vibration Reduction of Articulated Silicone Gel Column with Varying Geometry

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To improve vibration reduction effect in low-frequency band of dynamic vibration absorber (DVA), a novel type of articulated silicone gel column (SGC) is introduced in the design of the tuned dynamic vibration absorber. The nonlinear variation of frequency of SGC with varying geometry is obtained by both finite element simulation and experiments. The most sensitive mode is located, which has wider frequency range by varying the geometry. The polynomial fitting is used to describe nonlinear relation between frequency and geometry of the most sensitive mode. By tuning the geometry, the equivalent stiffness and then resonance frequencies can be manipulated to behave as an active vibration absorber. The vibration reduction experiment of SGC vibration absorbers is investigated. It is found that SGC has better vibration reduction effect in low-frequency band. The experimental results demonstrate that the vibration reduction effect can reach 94.03% when tuning SCG to the first order main resonance.

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Index Theory and Stability of Elliptic Relative Equilibria in Planar n-Body Problem

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It is well known that a planar central configuration of the n-body problem gives rise to solutions where each particle moves on a specific Keplerian orbit while the totality of the particles move on a homographic motion. Following Meyer and

Schmidt, we call such solutions elliptic relative equilibria. Some famous examples such as Lagrangian orbits, Euler orbits, etc. have important background in the solar system. In this talk, we will introduce several new methods to study its stability.

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Studying the Geometry and Dynamics of Prospective Parkinson-Inhibitor MCoCP4 Variants with Modified Grafting Topologies

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Cyclotides are special knotted proteins stabilized by three pairs of disulfide bonds. Onto such a molecule as the scaffolding, small peptides can be grafted, with some specific conformations and new functions emerging as a result of this grafting. Thus, there is experimental evidence that the molecule MCoCP4, obtained by grafting a linear derivative of the CP4 Parkinson inhibitor CLATWAVG onto loop 6 of the cyclotide MCoTI-I, can possibly reduce to a greater extent the cytotoxicity of α-synuclein and has therefore better chances as a therapeutic alternative against the Parkinson disease. In this study, we extend this grafting strategy to the cyclotide MCoTI-II, which is highly homologous with MCoTI-I. We analyze the dynamical and geometrical properties of this specific grafted molecule, as well as two variants with differing grafting topologies, using molecular dynamics (MD) simulations. By visualizing the local geometry in discrete Frenet Frames (DFF) along the trajectory, we analyze the impact of the grafting point on the behavior of the CP4-derived grafts. We quantify the backbone twisting and the sidechain orientation in all three scaffolds by calculating the folding index and the orientation of the Cβ atoms in the DFF formalism. For each grafting topology, we obtain a representative structure for the CP4-derived graft. We discuss the possibility of the obtained results giving rise to a ranking scheme aiming at optimisation of the desired biological effect of the so-engineered molecule.

This work was supported in part by China-Bulgaria Intergovernmental S&T Cooperation Project at the Ministry of Science and Technology of People's Republic of China (2021.1) and by the Bulgarian Science Fund under Grants KP-06-China-10/2020 and KP-06- COST-9/2019. Computational resources were provided by BioSim HPC cluster at the Faculty of Physics, St. Kliment Ohridski University of Sofia and by the Centre for Advanced Computing and Data Processing, supported under Grant BG05M2OP001-1.001-0003 by the Science and Education for Smart Growth Operational Program (2014-2020) and co-financed by the European Union through the European structural and investment funds.

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Parameter Optimization for Novel Dynamic Vibration Absorbers with Negative Stiffness and Amplifying Component

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Two novel dynamic vibration absorbers(DVA) with negative stiffness and amplifying mechanism are proposed. The dynamic equations are established and it is found that the models designed still have fixed points independent of damping in the amplitude frequency curves of the primary system. The optimum frequency ratio and approximate optimal damping ratio are respectively obtained based on the fixed point theory. To keep the stability of the systems, the optimum negative stiffness ratio and the best working range of inerter are deduced. Furthermore, the influence of different magnification ratio and inerter coefficient on the amplitude-frequency response is analyzed. At last, the control performances of the presented DVAs are compared with three existing typical DVAs, which show that the DVAs presented in this paper have better vibration reduction broadband effect. Moreover, the results of this paper can provide theoretical guidance for the installation position of inerter in the design of vibration absorbers of related structures.

Keywords: Dynamic vibration absorber, negative stiffness, fixed point theory, inerter-mass, lever component

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Traveling Waves of the KdV-nKdV Equation

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In this paper, the bifurcation theory of dynamical system is applied to study the traveling waves of the KdV-nKdV equation. By transforming the traveling wave system, we derive various parameter conditions, which guarantee the existence of its bounded and unbounded orbits. Furthermore, by calculating complicated elliptic integrals along these orbits, we obtain exact expressions of all possible traveling wave solutions of the KdV-nKdV equation.

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Review for Complex Nonlinear Dynamics and Vibration Suppression of Conceptual Airfoil Models

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During the past few decades, several significant progresses have been made in exploring complex nonlinear dynamics and vibration suppression of conceptual aeroelastic airfoil models. Additionally, some new challenges have arisen. To the best of the author's knowledge, most studies are concerned with the deterministic case, however, neglect the effects of stochasticity encountered in practical flight environments on the nonlinear dynamical behaviors of the airfoil systems. Crucially, coupling interaction of the structure nonlinearities and uncertainty fluctuations can lead to some difficulties on the airfoil models, including modeling, response solving, and vibration suppression. At the same time, most of the existing studies depend mainly on a mathematical model established by physical mechanisms. Unfortunately, it is challenging and even impossible to obtain an accurate physical model of the complex wing structure in engineering practice. The emergence of data science and machine learning provides new opportunities for understanding the aeroelastic airfoil systems from the data-driven point of view, such as data-driven modeling, prediction, and control from the recorded data. Nevertheless,

relevant data-driven problems of the aeroelastic airfoil systems are not addressed well up to now. This survey contributes to conducting a comprehensive overview of recent developments toward understanding complex dynamical behaviors and vibration suppression, especially for stochastic dynamics, early warning and data-driven problems, of the conceptual two-dimensional airfoil models with different structural nonlinearities. The results on the airfoil models are summarized and discussed. Besides, several potential development directions that are worth further exploring are also highlighted.

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Coverings and Nonlocal Symmetries As Well As Fundamental Solutions of Nonlinear Equations Derive from the Nonisospectral AKNS Hierarchy

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In this talk, we considered the positive-order and negative-order nonisospectral integrable AKNS hierarchies and some reductions, and further discussed a type of nonisospectral expanding integrable model, from which some reductions of equations are presented, including an interest rate model, a generalized bond price equation, etc. Finally, we studied their some properties, such as the Lie presentations and characteristic solutions, self-adjointness and conservation laws by using the Lie group theory.

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Modeling of Wave Propagation in a Viscoelastic Medium

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The propagation of waves provoked by a time dependant load in a hollow cylinder is studied by using various numerical approaches. The proposed semi-analytical

solution includes Laplace transform with respect to time applied to the wave equation accounting the specified stress-strain relationship, boundary, and initial conditions. The viscose behavior is superimposed to linear elasticity in terms of the linear Boltzmann-Volterra model. The solution is found in the space of images and is then 'projected back' into the space of originals. Alternatively, the wave propagation is investigated by monitoring the stress and the displacement evolution in a specified location within a transient, finite element analysis. Viscoelasticity is modeled by modifying the material parameters according to a predefined constitutive relation with the assumptions for isotropic viscoelasticity and constant Poisson's ratio. Both approaches employ the same geometry and define the same boundary conditions. The obtained results are compared and discussed.

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Averaging Principle for Monotone SPDEs

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The first Bogolyubov theorem on averaging for SDEs has been investigated extensively. In this talk, we will discuss the second Bogolyubov theorem and global averaging principle for monotone SPDEs.

This talk is based on our joint work with Mengyu Cheng.

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Recent Developments on Integrable Peakon Systems III: New Phenomenon – Weak-Kink and Its Interaction with Peakons

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In this talk, we speak about an integrable system with both quadratic and cubic nonlinearity. This model is kind of a cubic generalization of the Camassa-Holm (CH) equation. The equation is shown integrable with its Lax pair, bi-Hamiltonian structure, and infinitely many conservation laws. In the case of no linear term, the peaked soliton (peakon) and multi-peakon solutions are presented. In particular, the two-peakon dynamical system is explicitly presented and their collisions are investigated in details. In the case of keeping linear and cubic terms without quadratic term, the new phenomenon of weak kink and weak-kink-peakon interactional solutions are found. Significant difference from the CH equation is analyzed through a detailed comparison. In our paper, we also study all possible smooth soliton solutions for the proposed nonlinear system.

This work is joint with Baoqiang Xia and Jibin Li.

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